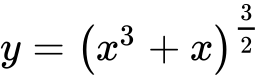
**C:\Users\User\Desktop\image001.gif**

**SUBJECT-MATH AA SL BASED ON TOPIC: CALCULUS**

**PUPIL NAME: ………………………………………. GRADE: XII M.M: 50**

**1a.** *[3 marks]*

Let .

Find .



**1b.** *[3 marks]*

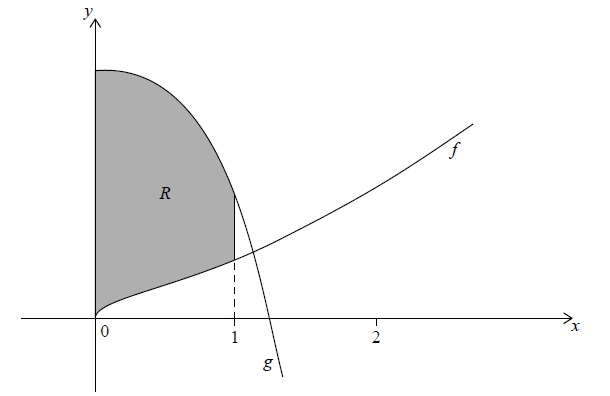
Hence find .



**1c.** *[3 marks]*

Consider the functions  and , for  ≥ 0.

The graphs of  and  are shown in the following diagram.



The shaded region  is enclosed by the graphs of , , the -axis and .

Write down an expression for the area of .



**1d.** *[6 marks]*

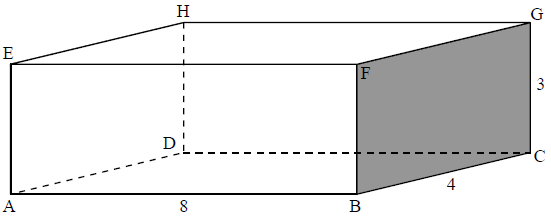
Hence find the exact area of .



**2a.** *[2 marks]*

The Happy Straw Company manufactures drinking straws.

The straws are packaged in small closed rectangular boxes, each with length 8 cm, width 4 cm and height 3 cm. The information is shown in the diagram.



Calculate the surface area of the box in cm2.

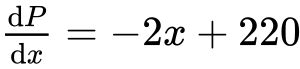


**2b.** *[2 marks]*

Calculate the length AG.



**2c.** *[3 marks]*

Each week, the Happy Straw Company sells  boxes of straws. It is known that ,  ≥ 0, where  is the weekly profit, in dollars, from the sale of  thousand boxes.

Find the number of boxes that should be sold each week to maximize the profit.



**2d.** *[5 marks]*

Find .



**2e.** *[3 marks]*

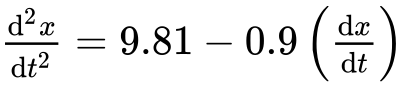
Find the least number of boxes which must be sold each week in order to make a profit.

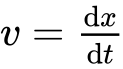


**3a.** *[7 marks]*

An object is placed into the top of a long vertical tube, filled with a thick viscous fluid, at time  seconds.

Initially it is thought that the resistance of the fluid would be proportional to the velocity of the object. The following model was proposed, where the object’s displacement, , from the top of the tube, measured in metres, is given by the differential equation

.

By substituting  into the equation, find an expression for the velocity of the particle at time . Give your answer in the form .



**3b.** *[2 marks]*

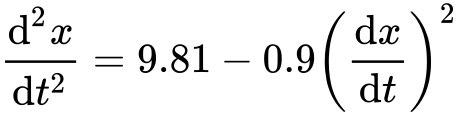
The maximum velocity approached by the object as it falls is known as the terminal velocity.

From your solution to part (a), or otherwise, find the terminal velocity of the object predicted by this model.



**3c.** *[2 marks]*

An experiment is performed in which the object is placed in the fluid on a number of occasions and its terminal velocity recorded. It is found that the terminal velocity was consistently smaller than that predicted by the model used. It was suggested that the resistance to motion is actually proportional to the velocity squared and so the following model was set up.



Write down the differential equation as a system of first order differential equations.



**3d.** *[4 marks]*

Use Euler’s method, with a step length of 0.2, to find the displacement and velocity of the object when .



**3e.** *[1 mark]*

By repeated application of Euler’s method, find an approximation for the terminal velocity, to five significant figures.



**3f.** *[2 marks]*

At terminal velocity the acceleration of an object is equal to zero.

Use the differential equation to find the terminal velocity for the object.



**3g.** *[2 marks]*

Use your answers to parts (d), (e) and (f) to comment on the accuracy of the Euler approximation to this model.

